Coffee tremors

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1 Introduction

Imagine you are a coffee addict, but that's not all, you have a big problem for a coffee addict: coffee produce you tremors.

You begin your day without coffee, and tremors are no problem because you haven't had any coffee yet. However, you go and drink 250 mL of coffee. You, as an addict to coffee, go immediately for your second coffee (also 250 mL). However, as you already drank coffee, you begin to tremor and spill part of your coffee, so you drink less than 250 mL.

2 Problem

Now suppose the intensity of the tremors depends on the volume of coffee you just drank. So, we can write a map equation that represents the volume you will drink in your next coffee (V_{n+1}) depending on the volume of coffee you drink in your current coffee (V_n):

$$V_{n+1} = V_0 - kV_n \tag{1}$$

where V_0 is the volume of coffee of the first coffee of the day and k is a constant that represents how much the coffee produces you tremors, so for example $k = 1$ indicates that if you drink 100 mL of coffee, you will spill 100 mL of your next coffee. kV_n would be the amount of coffee you spill due to tremors.

3 Analysis

3.1 Oscillations

In our problem, $V_n < 250$ mL and $0 < k < 1$. Suppose we have

$$V_{n+1} > V_n \tag{2}$$

Multiplying by $-k$, we get:

$$-kV_{n+1} < -kV_n$$

Then, adding V_0, we obtain:

$$V_0 - kV_{n+1} < V_0 - kV_n$$

thus,

$$V_{n+2} < V_{n+1} \tag{3}$$

From 2 and 3 we can see that this system shows oscillatory behaviour.

3.2 Fixed point

The fixed point is given when $V_{n+1} = V_n = V^*$.

$$V^* = \frac{V_0}{1 + k}$$

This fixed point indicates that after many cups of coffee you will end up spilling the same quantity each time. For example, if you put 250 mL in each cup of coffee and your $k = 0.5$, then after many cups of coffee, you will stabilize your tremors so that you drink

$$V^* = \frac{250}{1.5} = 166.\bar{6}\,\text{mL}$$